on Information and Systems

VOL. E104-D NO. 11
NOVEMBER 2021
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The Institute of Electronics, Information and Communication Engineers
Kikai-Shinko-Kaikan Bldg., 5-8, Shibakoen 3 chome, Minato-ku, TOKYO, 105-0011 JAPAN

# Provable-Security Analysis of Authenticated Encryption Based on Lesamnta-LW in the Ideal Cipher Model 

Shoichi HIROSE ${ }^{\dagger \mathrm{a}}$, Member, Hidenori KUWAKADO ${ }^{\dagger \dagger \mathrm{b})}$, Senior Member, and Hirotaka YOSHIDA ${ }^{\dagger \dagger \mathrm{c})}$, Member


#### Abstract

SUMMARY Hirose, Kuwakado and Yoshida proposed a nonce-based authenticated encryption scheme Lae0 based on Lesamnta-LW in 2019. Lesamnta-LW is a block-cipher-based iterated hash function included in the ISO/IEC 29192-5 lightweight hash-function standard. They also showed that Lae0 satisfies both privacy and authenticity if the underlying block cipher is a pseudorandom permutation. Unfortunately, their result implies only about 64-bit security for instantiation with the dedicated block cipher of Lesamnta-LW. In this paper, we analyze the security of Lae0 in the ideal cipher model. Our result implies about 120-bit security for instantiation with the block cipher of Lesamnta-LW. key words: authenticated encryption, hash function, Lesamnta-LW, ideal cipher model


## 1. Introduction

### 1.1 Background

Authenticated encryption (AE) is symmetric cryptography providing both privacy and authenticity. Informally, privacy is confidentiality of plaintexts and authenticity is integrity of ciphertexts. AE schemes often take additional input called associated data which only require authenticity. Such AE schemes are referred to as authenticated encryption with associated data (AEAD).

There are some kinds of approaches for AEAD construction. Among them, one of the most common approaches is to construct it as a mode of operation of a block cipher such as AES [1]. The other is to construct it based on the sponge construction [2]. The sponge construction [3] was invented originally for cryptographic hash functions as well as for MAC functions and stream ciphers. The spongebased hash function Keccak [4] was selected for the SHA-3 standard [5]. The sponge construction is also popular for lightweight hashing.

The ISO/IEC 29192-5 lightweight hash function standard [6] was released in 2016, which specifies three lightweight cryptographic hash functions: PHOTON [7],

[^0]SPONGENT [8], and Lesamnta-LW [9]. PHOTON and SPONGENT follow the sponge construction, and the sponge-based AEAD mode can be applied to them. On the other hand, Lesamnta-LW is a Merkle-Damgård [10], [11] hash function using a dedicated block cipher whose key size is half the block size as a compression function. In addition, Lesamnta-LW is optimized for software implementation, while both PHOTON and SPONGENT are optimized for hardware implementation. In fact, a software result [9] shows that Lesmanta-LW provides 120 -bit collision resistance with 54 bytes of RAM, achieving $20 \%$ faster shortmessage performance over SHA-256, while hardware results show that SPONGENT provides 80-bit collision resistance with 1329 GE and PHOTON provides the same security level with 1396 GE.

In 2019, Hirose, Kuwakado and Yoshida [12] proposed a nonce-based AEAD scheme based on LesamntaLW, which they called Lae0. It can be implemented with the block cipher of Lesamnta-LW. Thus, it is an efficient option for lightweight AEAD on low-cost 8-bit microcontrollers where RAM requirement is critical for cryptographic functionality.

### 1.2 Our Contribution

Hirose, Kuwakado and Yoshida [12] also showed that Lae0 is secure in the standard model: Lae0 satisfies both privacy and authenticity if the block cipher of the Lesamnta-LW hashing mode is a pseudorandom permutation (PRP). Unfortunately, their result is not entirely satisfactory in that it implies only about 64-bit security for instantiation of Lae0 with the block cipher of Lesamnta-LW. Their upper bound on the advantage of any adversary $\mathbf{A}$ against Lae0 has the term $\ell q \operatorname{adv}_{E}^{\mathrm{prp}}$, where $\operatorname{adv}_{E}^{\mathrm{prp}}$ is the advantage of an adversary constructed from $\mathbf{A}$ against the underlying block cipher $E, \ell$ is the maximum length of the queries made by $\mathbf{A}$, and $q$ is the number of the queries made by $\mathbf{A}$. Due to the simple key-guessing attack on $E, \operatorname{adv}_{E}^{\mathrm{prp}}=\Omega\left(t / 2^{w}\right)$, where $t$ is the run time of $\mathbf{A}$ and $w$ is the key length of $E$. Thus, the upper bound is $\Omega(1)$ if both $\ell q$ and $t$ are $\Omega\left(2^{w / 2}\right)$. For the block cipher of Lesamnta-LW, $w=128$.

In this paper, we analyze the security of Lae0 in the ideal cipher model. In terms of both privacy and authenticity, our result implies about 120-bit security for instantiation of Lae0 with the block cipher of Lesamnta-LW. We discuss the authenticity of LaeO under two typical misuses: nonce repetition (NR) and releasing unverified plaintexts (RUP).

Though our analysis assumes an ideal cipher, our result is still significant in that it implies security of LaeO against generic attacks regarding the underlying block cipher just as a black box.

### 1.3 Related Work

Authenticated encryption received the first formal treatments from Katz and Yung [13] and Bellare and Namprempre [14], which are followed by Jutla [15].

There are many block-cipher modes of operation for AEAD. OCB [16] is one of the earliest but most efficient modes, and it is inspired by IAPM [15]. CCM [17] and GCM [18] are specified by NIST and ISO/IEC 19772 [19].

As far as we know, there is only one proposal for AEAD based on cryptographic hashing except for the sponge-based proposals. It is OMD (Offset MerkleDamgård) by Cogliani et al. [20], which is a mode of operation of a compression function for the Merkle-Damgård hashing such as SHA-2 [21].

Nonce-based symmetric encryption was introduced with its formalization by Rogaway [22]. The generic composition of nonce-based AEAD was discussed by Namprempre et al. [23].

For misuse resistance of authenticated encryption, security under NR was formalized by Rogaway and Shrimpton [24]. Security under RUP was formalized by Andreeva et al. [25]. Robust authenticated encryption was introduced and formalized by Hoang et al. [26], which is secure under NR and RUP.

Improved and/or new security analyses of the Lesamnta-LW block cipher have recently been conducted by Hirose, Sasaki and Yoshida [27] and by Shiba et al. [28].

### 1.4 Organization

Notations and definitions used in the remaining parts are given in Sect. 2. Syntax and security are formalized for AEAD in Sect. 3. The nonce-based AEAD scheme Lae0 is described in Sect. 4. Lae0 is shown to satisfy both privacy and authenticity in Sect. 5. A brief concluding remark is given in Sect. 6.

## 2. Preliminaries

### 2.1 Notations

Let $\Sigma=\{0,1\}$. For any integer $l \geq 0$, let $\Sigma^{l}$ be identified with the set of all $\Sigma$-sequences of length $l . \Sigma^{0}=\{\varepsilon\}$, where $\varepsilon$ is the empty sequence. $\Sigma^{1}=\Sigma$. Let $\left(\Sigma^{l}\right)^{*}=\bigcup_{i \geq 0}\left(\Sigma^{l}\right)^{i}$ and $\left(\Sigma^{l}\right)^{+}=\bigcup_{i \geq 1}\left(\Sigma^{l}\right)^{i}$.

For $x \in \Sigma^{*}$, the length of $x$ is denoted by $|x|$. For $x_{1}, x_{2} \in \Sigma^{*}, x_{1} \| x_{2}$ represents their concatenation. For $x \in \Sigma^{*}$ and an integer $0 \leq l \leq|x|, \operatorname{msb}_{l}(x)$ represents the most significant $l$ bits of $x$, and $\operatorname{Isb}_{l}(x)$ represents the least significant $l$ bits of $x$.

For $x, y \in \Sigma^{*}$ such that $|x| \geq|y|$, let $x \oplus y$ and $y \oplus x$


Fig. 1 The hashing mode of Lesamnta-LW. The input of the block cipher $E$ from the top is its key input.
represent bitwise XOR of $x$ and $y \| 0^{|x|-|y|}$.
Selecting an element $s$ from a set $\mathcal{S}$ uniformly at random is denoted by $s \longleftarrow \mathcal{S}$.

The set of all functions from $\mathcal{X}$ to $\mathcal{Y}$ is denoted by $\mathcal{F}(\mathcal{X}, \boldsymbol{y})$. The set of all permutations on $\mathcal{X}$ is denoted by $\mathcal{P}(\mathcal{X})$. $\iota$ represents an identity permutation. The set of all block ciphers with a key size $\kappa$ and a block size $n$ is denoted by $\mathcal{B}(\kappa, n)$. A block cipher in $\mathcal{B}(\kappa, n)$ is called a $(\kappa, n)$ block cipher. For a keyed function $f: \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}, f(K, \cdot)$ is often denoted by $f_{K}(\cdot)$.

### 2.2 Hashing Mode of Lesamnta-LW

The hashing mode of Lesamnta-LW [9] is the plain MerkleDamgård iteration of a block cipher $E$ in $\mathcal{B}(n / 2, n)$, where $n$ is a positive even integer. $E$ works as a compression function with domain $\Sigma^{3 n / 2}$ and range $\Sigma^{n}$. It is depicted in Fig. 1. $I V_{0} \| I V_{1} \in \Sigma^{n}$ is an initialization vector, where $\left|I V_{0}\right|=\left|I V_{1}\right|=n / 2 . M_{1}, M_{2}, \ldots, M_{m}$ are message blocks, where $M_{i} \in \Sigma^{n / 2}$ for $i=1,2, \ldots, m$.

The dedicated block cipher of Lesamnta-LW is in $\mathcal{B}(128,256)$.

## 3. Authenticated Encryption with Associated Data

### 3.1 Syntax

A scheme of nonce-based authenticated encryption with associated data (AEAD) consists of a pair of functions for encryption and decryption. The encryption function is Enc : $\mathcal{K} \times \mathcal{N} \times \mathcal{A} \times \mathcal{M} \rightarrow \mathcal{C} \times \mathcal{T}$ and the decryption function is Dec: $\mathcal{K} \times \mathcal{N} \times \mathcal{A} \times \mathcal{C} \times \mathcal{T} \rightarrow \mathcal{M} \cup\{\perp\}$, where $\mathcal{K}$ is a key space, $\mathcal{N}$ is a nonce space, $\mathcal{A}$ is an associated-data space, $\mathcal{M}$ is a message space, $\mathcal{C}$ is a ciphertext space, and $\mathcal{T}$ is a tag space. $\mathcal{M} \subset \Sigma^{*}, \perp \notin \mathcal{M}$ and $\mathcal{A} \subset \Sigma^{*}$. If $M \in \mathcal{M}$, then $\Sigma^{|M|} \subset \mathcal{M}$. For any $K \in \mathcal{K}$, if $(C, T) \leftarrow \operatorname{Enc}_{K}(N, A, M)$ for some $(N, A, M) \in \mathcal{N} \times \mathcal{A} \times \mathcal{M}$, then $M \leftarrow \operatorname{Dec}_{K}(N, A, C, T)$. Otherwise, $\perp \leftarrow \operatorname{Dec}_{K}(N, A, C, T)$, which means that $(N, A, C, T)$ is invalid with respect to $K \in \mathcal{K}$.

### 3.2 Security

The security requirements for AEAD are privacy and authenticity. Informally, privacy is confidentiality of encrypted messages, and authenticity is integrity of ciphertexts and associated data.
(1) Privacy

Let $\$$ be a random function taking $(N, A, M) \in \mathcal{N} \times \mathcal{A} \times$
$\mathcal{M}$ as input and returning a binary sequence of length $\left|E c_{K}(N, A, M)\right|$, which is chosen uniformly at random. The privacy of an AEAD scheme (Enc, Dec) is defined by the indistinguishability between $\mathrm{Enc}_{K}$ and \$:

$$
\operatorname{Adv}_{(\text {Enc,Dec })}^{\text {priv }}(\mathbf{A})=\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{Enc} c_{K}}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\$}=1\right]\right|,
$$

where $K \longleftarrow \mathcal{K}$. $\mathbf{A}$ is assumed to be nonce-respecting. Namely, $\mathbf{A}$ is not allowed to make multiple encryption queries with the same nonce.

## (2) Authenticity

The authenticity of an AEAD scheme (Enc, Dec) is defined by the unforgeability:

$$
\operatorname{Adv}_{(\text {Enc,Dec })}^{\text {auth }}(\mathbf{A})=\operatorname{Pr}\left[\mathbf{A}^{\mathrm{Enc}_{K}, \mathrm{Dec}_{K}} \text { succeeds in forgery }\right]
$$

where $K \longleftrightarrow \mathcal{K}$. A succeeds in forgery if it succeeds in making a decryption query such that its corresponding reply from $\operatorname{Dec}_{K}$ is not $\perp$. $\mathbf{A}$ is not allowed to make a trivial decryption query. Namely, if $\mathbf{A}$ gets $(C, T)$ as an answer to some encryption query $(N, A, M)$, then it is not allowed to ask $(N, A, C, T)$ as a decryption query.

## 4. AEAD Based on Lesamnta-LW: Lae0

Let $E$ be a block cipher in $\mathcal{B}(n / 2, n)$, where $n$ is an even integer. Hereafter, let $n / 2=w$ just for simplicity.

The padding function used in the construction is defined as follows: For any $X \in \Sigma^{*}$,

$$
\operatorname{pad}(X)= \begin{cases}X & \text { if }|X|>0 \text { and }|X| \equiv 0 \quad(\bmod w) \\ X| | 10^{t} & \text { if }|X|=0 \text { or }|X| \equiv 0 \quad(\bmod w)\end{cases}
$$

where $t$ is the minimum non-negative integer satisfying $|X|+$ $1+t \equiv 0(\bmod w)$. For any $X \in \Sigma^{*},|\operatorname{pad}(X)|$ is the minimum positive multiple of $w$, which is greater than or equal to $|X|$. Notice that pad is not injective. For example, $\operatorname{pad}(\varepsilon)=$ $\operatorname{pad}\left(10^{\omega-1}\right)=10^{\omega-1}$.

Let $\operatorname{pad}(X)=\left(X_{1}, X_{2}, \ldots, X_{x}\right)$, where $\left|X_{i}\right|=w$ for every $i$ such that $1 \leq i \leq x$. $x=1$ if $|X|=0$, and $x=\lceil|X| / w\rceil$ if $|X|>0 . X_{i}$ is called the $i$-th block of $\operatorname{pad}(X)$.

For $E \in \mathcal{B}(w, n)$ and $\pi_{0}, \pi_{1} \in \mathcal{P}\left(\Sigma^{w}\right)$, the nonce-based AEAD scheme Lae0 $=(\mathrm{E} 0, \mathrm{DO})$ is presented by Algorithm 1. The encryption function EO is also depicted in Fig. 2. For Lae0, the key space is $\Sigma^{w}$, the nonce space is $\Sigma^{n}$,
and the tag space is $\Sigma^{\tau}$, where $0<\tau \leq n$. The associateddata space, the message space and the ciphertext space are $\Sigma^{*}$. If $(C, T) \leftarrow \mathrm{EO}_{K}(N, A, M)$, then $|C|=|M|$.

```
Algorithm 1 Encryption E0 and decryption D0 of Lae0
    function \(\mathrm{EO}_{K}(N, A, M)\)
        \(\left(A_{1}, A_{2}, \ldots, A_{a}\right) \leftarrow \operatorname{pad}(A) ;\)
        \(\left(M_{1}, M_{2}, \ldots, M_{m}\right) \leftarrow \operatorname{pad}(M) ;\)
        \(Y_{0} \leftarrow E_{K}(N) ;\)
        for \(i=1\) to \(a-1\) do \(\quad \triangleright Y_{i-1,0}=\operatorname{msb}_{w}\left(Y_{i-1}\right), Y_{i-1,1}=\operatorname{lsb}_{w}\left(Y_{i-1}\right)\)
            \(Y_{i} \leftarrow E_{Y_{i-1,0}}\left(A_{i} \| Y_{i-1,1}\right) ;\)
        if \(|A|>0 \wedge|A| \equiv 0(\bmod w)\) then
            \(Y_{a} \leftarrow E_{Y_{a-1,0}}\left(A_{a} \| \pi_{0}\left(Y_{a-1,1}\right)\right) ;\)
        else
            \(Y_{a} \leftarrow E_{Y_{a-1,0}}\left(A_{a} \| \pi_{1}\left(Y_{a-1,1}\right)\right) ;\)
        for \(i=1\) to \(m-1\) do
            \(C_{i} \leftarrow M_{i} \oplus Y_{a+i-1,1} ;\)
            \(Y_{a+i} \leftarrow E_{Y_{a+i-1,0}}\left(M_{i \|} \| Y_{a+i-1,1}\right) ;\)
        \(C_{m} \leftarrow M_{m} \oplus Y_{a+m-1,1} ;\)
        if \(|M|>0 \wedge|M| \equiv 0(\bmod w)\) then
            \(T \leftarrow E_{Y_{a+m-1,0}}\left(M_{m} \| \pi_{0}\left(Y_{a+m-1,1}\right)\right) ;\)
        else
            \(T \leftarrow E_{Y_{a+m-1,0}}\left(M_{m} \| \pi_{1}\left(Y_{a+m-1,1}\right)\right) ;\)
        \(C \leftarrow C_{1}\|\cdots\| C_{m-1} \| \mathrm{msb}_{|M|-(m-1) \omega}\left(C_{m}\right) ;\)
        return \(C, T\);
    function \(\mathrm{DO}_{K}(N, A, C, T)\)
        \(\left(A_{1}, A_{2}, \ldots, A_{a}\right) \leftarrow \operatorname{pad}(A) ;\)
        \(\left(C_{1}, C_{2}, \ldots, C_{m}\right) \leftarrow \operatorname{pad}(C) ;\)
        \(Y_{0} \leftarrow E_{K}(N) ;\)
        for \(i=1\) to \(a-1\) do
            \(Y_{i} \leftarrow E_{Y_{i-1,0}}\left(A_{i} \| Y_{i-1,1}\right) ;\)
        if \(|A|>0 \wedge|A| \equiv 0(\bmod w)\) then
            \(Y_{a} \leftarrow E_{Y_{a-1,0}}\left(A_{a} \| \pi_{0}\left(Y_{a-1,1}\right)\right) ;\)
        else
            \(Y_{a} \leftarrow E_{Y_{a-1,0}}\left(A_{a} \| \pi_{1}\left(Y_{a-1,1}\right)\right) ;\)
        for \(i=1\) to \(m-1\) do
            \(M_{i} \leftarrow C_{i} \oplus Y_{a+i-1,1} ;\)
            \(Y_{a+i} \leftarrow E_{Y_{a+i-1,0}}\left(M_{i} \| Y_{a+i-1,1}\right) ;\)
        \(M_{m} \leftarrow C_{m} \oplus \operatorname{msb}_{|C|-(m-1) w}\left(Y_{a+m-1,1}\right) ;\)
        if \(|C|>0 \wedge|C| \equiv 0(\bmod w)\) then
            \(T^{\prime} \leftarrow E_{Y_{a+m-1,0}}\left(M_{m} \| \pi_{0}\left(Y_{a+m-1,1}\right)\right) ;\)
        else
            \(T^{\prime} \leftarrow E_{Y_{a+m-1,0}}\left(M_{m} \| \pi_{1}\left(Y_{a+m-1,1}\right)\right) ;\)
        \(M \leftarrow M_{1}\|\cdots\| M_{m-1} \| \mathrm{msb}_{|C|-(m-1) \omega}\left(M_{m}\right) ;\)
        if \(T^{\prime}=T\) then
            return \(M\);
        else
            return \(\perp\);
```



Fig. 2 The encryption function E0 of the nonce-based AEAD scheme Lae0. $(C, T) \leftarrow E 0_{K}(N, A, M)$, where $\operatorname{pad}(A)=\left(A_{1}, A_{2}\right), \operatorname{pad}(M)=\left(M_{1}, M_{2}, M_{3}\right)$, and $C=C_{1}\left\|C_{2}\right\| C_{3}$. This figure assumes that $|A| \not \equiv 0(\bmod w),|M| \equiv 0(\bmod w)$ and $\tau=n$.

## 5. Security of Lae0 in the Ideal Cipher Model

The security of LaeO $=(E 0, D 0)$ is analyzed in the ideal cipher model. Thus, adversaries are given oracle access to encryption $E$ and decryption $E^{-1}$ of the block cipher used in Lae0. Without loss of generality, it is assumed that adversaries do not make trivial queries to $E$ and $E^{-1}$. Namely, once an adversary obtains a triplet ( $S, U, V$ ) such that $E_{S}(U)=V$ by a query to $E$ or $E^{-1}$, it makes no new queries on the triplet.

A combinatorial theorem used in the analysis is first presented:

Lemma 1 (Theorem 3.1 in [29]) Suppose that there are $t$ balls and $t$ bins and that each ball is placed in a bin chosen independently and uniformly at random. Then, with probability at least $1-1 / t$, no bin has more than e $\ln t / \ln \ln t$ balls in it.

For Lemma 1, let $t=2^{w}$. Then,

$$
\mathrm{e} \ln t / \ln \ln t=\mathrm{e} w /\left(\log _{2} w-\log _{2} \log _{2} \mathrm{e}\right)
$$

which is denoted by $\gamma(w)$ in the remaining part.
Example $1 \gamma(128) \approx 53.77$.

### 5.1 Privacy

From the theorem given below, for privacy, Lae0 is secure against nonce-respecting adversaries causing at most $O\left(2^{w} / \gamma(w)\right)$ evaluations of its underlying block cipher. For $w=128,2^{w} / \gamma(w) \approx 2^{122.25}$.

The main idea of the proof of the following theorem is simple: The outputs of EO look random to an adversary if the set of the triplets $(S, U, V)$ such that $E_{S}(U)=V$ used by the process of E0 and the set of them obtained by the queries to $E$ and $E^{-1}$ made by the adversary are disjoint.

Theorem 1 Let $\mathbf{A}$ be any adversary against LaeO for privacy. Suppose that A makes at most $q_{\mathrm{e}}$ and $q_{\mathrm{d}}$ queries to $E$ and $E^{-1}$, respectively. Suppose that $\sigma$ is the total number of the queries to $E$ induced by the queries to EO and DO made by $\mathbf{A}$. Let $q=q_{\mathrm{e}}+q_{\mathrm{d}}$ and suppose that $q+\sigma \leq 2^{w}$. Then,

$$
\operatorname{Adv}_{\text {Lae } 0}^{\text {priv }}(\mathbf{A}) \leq \frac{\gamma(w) q_{\mathrm{e}}+q_{\mathrm{d}}+q+\sigma+1}{2^{w}}+\frac{(q+\sigma)^{2}}{2^{n-1}}
$$

in the ideal cipher model.
Proof This proof uses the game transformation technique. In the game PGr1 given in Fig. 3, BCenc and BCdec implement $E$ and $E^{-1}$ using lazy evaluation, respectively, and AEenc implements EO. Thus,

$$
\operatorname{Pr}\left[\mathbf{A}^{\mathrm{E} 0_{K}}=1\right]=\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 1}=1\right] .
$$

PGr2 differs from PGr1 only in $\mathscr{E}$ and $\mathscr{D}$, which are

| Initialization: |  |
| :---: | :---: |
| 100: $K \longleftarrow \Sigma^{w}$; |  |
| 101: $\mathrm{E}[S, U] \leftarrow \perp$ for ever |  |
| 102: $\mathrm{D}[S, V] \leftarrow \perp$ for ever |  |
| 103: $P_{S} \leftarrow\{ \}$ for every $S$; <br> 104: bad $\leftarrow$ false; | for every $S$; |
| Function $\mathscr{E}(S, U)$ : | Function $\mathscr{D}(S, V)$ : |
| 200: if $\mathrm{E}[S, U]=\perp$ then | 300: if $\mathrm{D}[S, V]=\perp$ then |
| 201: $\quad V \longleftarrow \Sigma^{n} \backslash C_{S}$ | 301: $\quad U \longleftarrow \Sigma^{n} \backslash P_{S}$ |
| 202: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ | 302: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ |
| 203: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ | 303: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ |
| 204: $\mathrm{E}[S, U] \leftarrow V$ | 304: $\mathrm{E}[S, U] \leftarrow V$ |
| 205: $\mathrm{D}[S, V] \leftarrow U$ | 305: $\mathrm{D}[S, V] \leftarrow U$ |
| 206: return $\mathrm{E}[S, U]$ | 306: return $\mathrm{D}[S, V]$ |

Oracle $\operatorname{AEenc}(N, A, M)$ :

```
500:(A},\mp@subsup{A}{1}{},\mp@subsup{A}{2}{},\ldots,\mp@subsup{A}{a}{})\leftarrow\operatorname{pad}(A)
```

501: $\left(M_{1}, M_{2}, \ldots, M_{m}\right) \leftarrow \operatorname{pad}(M)$;
502: $Y_{0} \leftarrow \mathscr{E}(K, N)$;
503: for $i=1$ to $a-1$ do
504: $\quad Y_{i} \leftarrow \mathscr{E}\left(Y_{i-1,0}, A_{i} \| Y_{i-1,1}\right)$;
505: if $|A|>0 \wedge|A| \equiv 0(\bmod w)$ then
506: $\quad Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{0}\left(Y_{a-1,1}\right)\right)$;
507: else
508: $\quad Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{1}\left(Y_{a-1,1}\right)\right)$;
509: for $i=1$ to $m-1$ do
$C_{i} \leftarrow M_{i} \oplus Y_{a+i-1,1} ;$
$Y_{a+i} \leftarrow \mathscr{E}\left(Y_{a+i-1,0}, M_{i} \| Y_{a+i-1,1}\right) ;$
$C_{m} \leftarrow M_{m} \oplus Y_{a+m-1,1} ;$
if $|M|>0 \wedge|M| \equiv 0(\bmod w)$ then
$T \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{0}\left(Y_{a+m-1,1}\right)\right) ;$
else
$T \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{1}\left(Y_{a+m-1,1}\right)\right) ;$
17: $C \leftarrow C_{1}\|\cdots\| C_{m-1} \| \mathrm{msb}_{|M|-(m-1) w}\left(C_{m}\right)$;
518: return $C, T$;

| Oracle BCenc $(S, U):$ | Oracle $\mathrm{BCdec}(S, V)$ : |
| :--- | :--- |
| 600: return $\mathscr{E}(S, U) ;$ | $600:$ return $\mathscr{D}(S, V) ;$ |

Fig. 3 Game PGr1

| Function $\mathscr{E}(S, U)$ : | Function $\mathscr{D}(S, V)$ : |
| :---: | :---: |
| 200: if $\mathrm{E}[S, U]=\perp$ then | 300: if $\mathrm{D}[S, V]=\perp$ then |
| 201: $\quad V \leftrightarrow \Sigma^{n}$ | 301: $\quad U \longleftarrow \Sigma^{n}$ |
| 202: if $V \in C_{S}$ then | 302: $\quad$ if $U \in P_{S}$ then |
| 203: $\quad$ bad $\leftarrow$ true | 303: $\quad$ bad $\leftarrow$ true |
| 204: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ | 304: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ |
| 205: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ | 305: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ |
| 206: $\mathrm{E}[S, U] \leftarrow V$ | 306: $\mathrm{E}[S, U] \leftarrow V$ |
| 207: $\mathrm{D}[S, V] \leftarrow U$ | 307: $\mathrm{D}[S, V] \leftarrow U$ |
| 208: return $\mathrm{E}[S, U]$ | 308: return $\mathrm{D}[S, V]$ |

Fig. 4 Game PGr2
given in Fig.4. PGr2 is equivalent to PGr1 until bad gets true in $\mathscr{E}$ or $\mathscr{D}$. Thus,

$$
\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 1}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 2}=1\right]\right| \leq(q+\sigma)^{2} / 2^{n+1}
$$

PGr3 differs from PGr2 only in Initialization, $\mathscr{E}$ and $\mathscr{D}$, which are given in Fig. 5. The differences are minor, and

$$
\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 3}=1\right]=\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 2}=1\right] .
$$

PGr4 differs from PGr3 only in $\mathscr{E}$ and $\mathscr{D}$, which are

## Initialization:

```
100:K}\leftarrow\mp@subsup{\Sigma}{}{w}\mathrm{ ;
101: }\textrm{E}[S,U]\leftarrow\perp\mathrm{ for every (S,U);
102: D[S,V]\leftarrow\perp for every (S,V);
103: bad }\leftarrow\mathrm{ false;
```

| Function $\mathscr{E}(S, U):$ | Function $\mathscr{D}(S, V)$ : |
| :--- | :--- |
| 200: if $\mathrm{E}[S, U]=\perp$ then | 300: if $\mathrm{D}[S, V]=\perp$ then |
| 201: $\quad V \leftarrow \Sigma^{n}$ | $301: \quad U \overleftrightarrow{\Sigma^{n}}$ |
| 202: $\mathrm{E}[S, U] \leftarrow V$ | $302: \quad \mathrm{E}[S, U] \leftarrow V$ |
| 203: $\mathrm{D}[S, V] \leftarrow U$ | 303: $\mathrm{D}[S, V] \leftarrow U$ |
| 204: return $\mathrm{E}[S, U]$ | 304: return $\mathrm{D}[S, V]$ |

Fig. 5 Game PGr3

| Function $\mathscr{E}(S, U):$ | Function $\mathscr{D}(S, V):$ |
| :--- | :--- |
| 200: if $\mathrm{E}[S, U] \neq \perp$ then | 300: if $\mathrm{D}[S, V] \neq \perp$ then |
| 201: $\quad b a d \leftarrow$ true | 301: $\quad b a d \leftarrow$ true |
| 202: else | 302: else |
| 203: $V \leftarrow \Sigma^{n}$ | 303: $U \leftarrow \Sigma^{n}$ |
| 204: $\mathrm{E}[S, U] \leftarrow V$ | 304: $\mathrm{E}[S, U] \leftarrow V$ |
| 205: $\mathrm{D}[S, V] \leftarrow U$ | 305: $\mathrm{D}[S, V] \leftarrow U$ |
| 206: return $\mathrm{E}[S, U]$ | 306: return $\mathrm{D}[S, V]$ |

Fig. 6 Game PGr4
given in Fig. 6. The differences are also minor, and

$$
\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 4}=1\right]=\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 3}=1\right]
$$

In the game PGil given in Fig. 7, BCenc and BCdec implement $E$ and $E^{-1}$ using lazy evaluation, respectively, and AEenc implements $\$$. Thus,

$$
\operatorname{Pr}\left[\mathbf{A}^{\$}=1\right]=\operatorname{Pr}\left[\mathbf{A}^{\text {PGil }}=1\right] .
$$

PGi2 differs from PGi1 only in $\mathscr{E}$ and $\mathscr{D}$, which are given in Fig. 8. Similar to the transformation from PGr1 to PGr3,

$$
\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 1}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 2}=1\right]\right| \leq q^{2} / 2^{n+1}
$$

Notice that $\mathscr{E}$ and $\mathscr{D}$ are called only by BCenc and BCdec, respectively.

PGr 4 is equivalent to PGi 2 until bad gets true in PGr 4 . Let Bad be the event that $\mathbf{A}^{\text {PGr4 }}$ sets bad true. Then,

$$
\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 4}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 2}=1\right]\right| \leq \operatorname{Pr}[\mathrm{Bad}] .
$$

For PGr4, let Hit be the event that $\mathscr{E}$ receives a query $(K, U)$ for some $U$ except for the cases that $(K, N)$ is the first query made by AEenc to respond to a query $(N, A, M)$ made by $\mathbf{A}$, or $\mathscr{D}$ receives a query $(K, V)$ for some $V$. Then,

$$
\operatorname{Pr}[\mathrm{Bad}] \leq \operatorname{Pr}[\mathrm{Hit}]+\operatorname{Pr}[\mathrm{Bad} \mid \overline{\mathrm{Hit}}]
$$

and

$$
\operatorname{Pr}[\mathrm{Hit}] \leq(q+\sigma) / 2^{w} .
$$

For Bad, let $\mathrm{Bad}_{\mathrm{AE}}$ be the event that a query from AEenc to $\mathscr{E}$ sets $b a d$ true for the first time and $\mathrm{Bad}_{\mathrm{BC}}$ be the event that a query from BCenc or BCdec sets bad true for the

| Initialization: |  |
| :---: | :---: |
| 100: $\mathrm{E}[S, U] \leftarrow \perp$ for every $(S, U)$; |  |
| 101: $\mathrm{D}[S, V] \leftarrow \perp$ for every $(S, V)$; |  |
| 102: $P_{S} \leftarrow\{ \}$ for every $S$; | for every $S$; |
| Function $\mathscr{E}(S, U)$ : | Function $\mathscr{D}(S, V)$ : |
| 200: if $\mathrm{E}[S, U]=\perp$ then | 300: if $\mathrm{D}[S, V]=\perp$ then |
| 201: $\quad V \longleftrightarrow \Sigma^{n} \backslash C_{S}$ | 301: $\quad U \longleftarrow \Sigma^{n} \backslash P_{S}$ |
| 202: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ | 302: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ |
| 203: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ | 303: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ |
| 204: $\mathrm{E}[S, U] \leftarrow V$ | 304: $\mathrm{E}[S, U] \leftarrow V$ |
| 205: $\mathrm{D}[S, V] \leftarrow U$ | 305: $\mathrm{D}[S, V] \leftarrow U$ |
| 206: return $\mathrm{E}[S, U]$ | 306: return $\mathrm{D}[S, V]$ |

Oracle AEenc $(N, A, M)$ :
500: $C \longleftarrow \Sigma^{|M|} ; T \longleftarrow \Sigma^{n} ;$
501: return $C, T$;
Oracle $\mathrm{BCenc}(S, U)$ : Oracle $\mathrm{BCdec}(S, V)$ :
600: return $\mathscr{E}(S, U)$;
600: return $\mathscr{D}(S, V)$;
Fig. 7 Game PGi1

| Function $\mathscr{E}(S, U):$ | Function $\mathscr{D}(S, V):$ |
| :--- | :--- |
| 200: $V \longleftarrow \Sigma^{n}$ | 300: $U \longleftarrow \leftarrow \Sigma^{n}$ |
| 201: $\mathrm{E}[S, U] \leftarrow V$ | 301: $\mathrm{E}[S, U] \leftarrow V$ |
| 202: $\mathrm{D}[S, V] \leftarrow U$ | 302: $\mathrm{D}[S, V] \leftarrow U$ |
| 203: return $\mathrm{E}[S, U]$ | 303: return $\mathrm{D}[S, V]$ |

Fig. 8 Game PGi2
first time. Then,

$$
\begin{aligned}
& \operatorname{Pr}[\mathrm{Bad} \mid \overline{\mathrm{Hit}}] \leq \operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}} \mid \overline{\mathrm{Hit}}\right]+\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BC}} \mid \overline{\mathrm{Hit}}\right] . \\
& \operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}} \mid \overline{\mathrm{Hit}}\right] \leq \sigma(q+\sigma) / 2^{n} .
\end{aligned}
$$

Further, for $\mathrm{Bad}_{\mathrm{BC}}$, let $\mathrm{Bad}_{\mathrm{BCe}}$ and $\mathrm{Bad}_{\mathrm{BCd}}$ be the events that a query from BCenc and BCdec sets bad true for the first time, respectively. Then,

$$
\operatorname{Pr}\left[\operatorname{Bad}_{\mathrm{BC}} \mid \overline{\mathrm{Hit}}\right] \leq \operatorname{Pr}\left[\operatorname{Bad}_{\mathrm{BCe}} \mid \overline{\mathrm{Hit}}\right]+\operatorname{Pr}\left[\operatorname{Bad}_{\mathrm{BCd}} \mid \overline{\mathrm{Hit}}\right] .
$$

For Bad $_{\mathrm{BCe}}$, from Lemma 1,

$$
\operatorname{Pr}\left[\operatorname{Bad}_{\mathrm{BCe}} \mid \overline{\mathrm{Hit}}\right] \leq \gamma(w) q_{\mathrm{e}} / 2^{w}+1 / 2^{w}
$$

For $\mathrm{Bad}_{\mathrm{BCd}}$, considering the probability of collision among the replies from $\mathscr{E}$, we obtain

$$
\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCd}} \mid \overline{\mathrm{Hit}}\right] \leq q_{\mathrm{d}} / 2^{w}+(q+\sigma)^{2} / 2^{n+1}
$$

Thus,

$$
\begin{aligned}
& \left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 4}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 2}=1\right]\right| \\
& \quad \leq \frac{\gamma(w) q_{\mathrm{e}}+q_{\mathrm{d}}+q+\sigma+1}{2^{w}}+\frac{\sigma(q+\sigma)}{2^{n}}+\frac{(q+\sigma)^{2}}{2^{n+1}} .
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\operatorname{Adv}_{\text {Lae0 }}^{\text {priv }}(\mathbf{A}) \leq \mid & \operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 4}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 2}=1\right] \mid \\
& +\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 1}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGr} 2}=1\right]\right| \\
& +\left|\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 1}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\mathrm{PGi} 2}=1\right]\right|
\end{aligned}
$$

$$
\begin{aligned}
& \leq \frac{\gamma(w) q_{\mathrm{e}}+q_{\mathrm{d}}+q+\sigma+1}{2^{w}} \\
& \quad+\frac{\sigma(q+\sigma)}{2^{n}}+\frac{(q+\sigma)^{2}}{2^{n}}+\frac{q^{2}}{2^{n+1}},
\end{aligned}
$$

where

$$
\frac{\sigma(q+\sigma)}{2^{n}}+\frac{(q+\sigma)^{2}}{2^{n}}+\frac{q^{2}}{2^{n+1}} \leq \frac{(q+\sigma)^{2}}{2^{n-1}}
$$

### 5.2 Authenticity

We discuss the authenticity of Lae0 under misuses. Namely, we assume NR and RUP in the following analysis.

Definition 1 Let $\Pi \subset \mathcal{P}(\mathcal{X})$. We say that $\Pi$ is pairwise everywhere distinct if, for every $\pi, \pi^{\prime} \in \Pi$ such that $\pi \neq \pi^{\prime}$, $\pi(x) \neq \pi^{\prime}(x)$ for every $x \in \mathcal{X}$.

From the following theorem, for authenticity, Lae0 is secure against adversaries causing at most $O\left(2^{w} / \gamma(w)\right)$ evaluations of its underlying block cipher in the setting allowing both NR and RUP.

Theorem 2 For permutations $\pi_{0}$ and $\pi_{1}$ on $\Sigma^{w}$ used in Lae0, suppose that $\left\{\pi_{0}, \pi_{1}, \iota\right\}$ is pairwise everywhere distinct. Let A be any adversary against Lae0 for authenticity. Suppose that $\mathbf{A}$ makes at most $q_{\mathrm{e}}$ and $q_{\mathrm{d}}$ queries to $E$ and $E^{-1}$, respectively, and $q_{\mathrm{D}}$ queries to D 0 . Suppose that $\sigma$ is the total number of the queries to $E$ induced by the queries to $E 0$ and D 0 made by $\mathbf{A}$. Let $q=q_{\mathrm{e}}+q_{\mathrm{d}}$ and suppose that $q+\sigma \leq 2^{w}$. Then,

$$
\operatorname{Adv}_{\mathrm{Lae} 0}^{\text {auth }}(\mathbf{A}) \leq \frac{3 \gamma(w) q+q_{\mathrm{d}}}{2^{w}-1}+\frac{q+\sigma+1}{2^{w}}+\frac{q_{\mathrm{D}}+7 \sigma^{2}+3 q \sigma}{2^{n}-q-\sigma}
$$

in the ideal cipher model.
Proof In this proof, we refer to the game AG1 given in Fig. 9. In this game, BCenc and BCdec implement $E$ and $E^{-1}$ using lazy evaluation, respectively. AEenc and AEdec implement E0 and DO, respectively.

It is assumed that, for each query made by $\mathbf{A}$, AEenc and AEdec give all $Y_{j, 1}$ 's to $\mathbf{A}$ together with the corresponding reply. Then, they are more informative than in the RUP setting.

Let $\mathcal{E}$ be the set of input-output pairs of the underlying block cipher obtained by the queries to $\mathscr{E}$ induced by the queries to AEenc and AEdec.

Let MCol be the event that

$$
\max _{v \in \Sigma^{w}}\left|\left\{(S, U, V) \in \mathcal{E} \mid \operatorname{lsb}_{w}(V)=v\right\}\right|>\gamma(w) .
$$

Then, since $\sigma \leq 2^{w}$, from Lemma 1 ,
$\operatorname{Pr}[\mathrm{MCOI}] \leq 1 / 2^{\omega}$.
For a query to $\mathscr{E}$ or $\mathscr{D}$, let $\mathcal{W}_{\mathrm{AE}}$ be the set of $(S, U, V)$ 's

## Initialization:

| 100: $K \longleftarrow \leftarrow \Sigma^{w} ;$ |  |
| :--- | :--- |
| 101: $\mathrm{E}[S, U] \leftarrow \perp$ for every $(S, U) ;$ |  |
| 102: $\mathrm{D}[S, V] \leftarrow \perp$ for every $(S, V) ;$ |  |
| 103: $P_{S} \leftarrow\{ \} ; C_{S} \leftarrow\{ \} ;$ | Function $\mathscr{D}(S, V):$ |
| Function $\mathscr{E}(S, U):$ | 300: if $\mathrm{D}[S, V]=\perp$ then |
| 200: if $\mathrm{E}[S, U]=\perp$ then | 301: $U \leftarrow \Sigma^{n} \backslash P_{S}$ |
| 201: $V \leftarrow \Sigma^{n} \backslash C_{S}$ | 302: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ |
| 202: $\quad P_{S} \leftarrow P_{S} \cup\{U\}$ | 303: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ |
| 203: $\quad C_{S} \leftarrow C_{S} \cup\{V\}$ | 304: $\mathrm{E}[S, U] \leftarrow V$ |
| 204: $\mathrm{E}[S, U] \leftarrow V$ | 305: $\mathrm{D}[S, V] \leftarrow U$ |
| 205: $\quad \mathrm{D}[S, V] \leftarrow U$ | 306: return $\mathrm{D}[S, V]$ |
| 206: return $\mathrm{E}[S, U]$ |  |

## Oracle AEenc $(N, A, M)$ :

500: $\left(A_{1}, A_{2}, \ldots, A_{a}\right) \leftarrow \operatorname{pad}(A)$;
501: $\left(M_{1}, M_{2}, \ldots, M_{m}\right) \leftarrow \operatorname{pad}(M)$;
502: $Y_{0} \leftarrow \mathscr{E}(K, N)$;
503: for $i=1$ to $a-1$ do
504: $\quad Y_{i} \leftarrow \mathscr{E}\left(Y_{i-1,0}, A_{i} \| Y_{i-1,1}\right)$;
505: if $|A|>0 \wedge|A| \equiv 0(\bmod w)$ then
6: $\quad Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{0}\left(Y_{a-1,1}\right)\right) ;$
else
$Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{1}\left(Y_{a-1,1}\right)\right) ;$
for $i=1$ to $m-1$ do
$C_{i} \leftarrow M_{i} \oplus Y_{a+i-1,1} ;$
$Y_{a+i} \leftarrow \mathscr{E}\left(Y_{a+i-1,0}, M_{i} \| Y_{a+i-1,1}\right) ;$
$C_{m} \leftarrow M_{m} \oplus Y_{a+m-1,1} ;$
if $|M|>0 \wedge|M| \equiv 0(\bmod w)$ then
$T \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{0}\left(Y_{a+m-1,1}\right)\right) ;$
else
$T \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{1}\left(Y_{a+m-1,1}\right)\right) ;$
$C \leftarrow C_{1}\|\cdots\| C_{m-1} \| \mathrm{msb}_{|M|-(m-1) w}\left(C_{m}\right) ;$
return $C, T$;
Oracle $\operatorname{AEdec}(N, A, C, T)$ :
600: $\left(A_{1}, A_{2}, \ldots, A_{a}\right) \leftarrow \operatorname{pad}(A)$;
601: $\left(C_{1}, C_{2}, \ldots, C_{m}\right) \leftarrow \operatorname{pad}(C)$;
602: $Y_{0} \leftarrow \mathscr{E}(K, N)$;
603: for $i=1$ to $a-1$ do
604: $\quad Y_{i} \leftarrow \mathscr{E}\left(Y_{i-1,0}, A_{i} \| Y_{i-1,1}\right)$;
605: if $|A|>0 \wedge|A| \equiv 0(\bmod w)$ then
606: $\quad Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{0}\left(Y_{a-1,1}\right)\right)$;
607: else
608: $\quad Y_{a} \leftarrow \mathscr{E}\left(Y_{a-1,0}, A_{a} \| \pi_{1}\left(Y_{a-1,1}\right)\right)$;
609: for $i=1$ to $m-1$ do
610: $\quad M_{i} \leftarrow C_{i} \oplus Y_{a+i-1,1}$;
611: $\quad Y_{a+i} \leftarrow \mathscr{E}\left(Y_{a+i-1,0}, M_{i} \| Y_{a+i-1,1}\right)$;
612: $M_{m} \leftarrow C_{m} \oplus \mathrm{msb}_{|C|-(m-1) w}\left(Y_{a+m-1,1}\right)$;
613: if $|C|>0 \wedge|C| \equiv 0(\bmod w)$ then
614: $\quad T^{\prime} \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{0}\left(Y_{a+m-1,1}\right)\right)$;
615: else
616: $\quad T^{\prime} \leftarrow \mathscr{E}\left(Y_{a+m-1,0}, M_{m} \| \pi_{1}\left(Y_{a+m-1,1}\right)\right)$;
617: $M \leftarrow M_{1}\|\cdots\|_{m-1} \| \mathrm{msb}_{|C|-(m-1) w}\left(M_{m}\right)$;
618: if $T^{\prime}=T$ then
619: return $M$;
620: else
621: return $\perp$;

| Oracle BCenc $(S, U):$ | Oracle $\operatorname{BCdec}(S, V)$ : |
| :--- | :--- |
| 600: return $\mathscr{E}(S, U) ;$ | $600:$ return $\mathscr{D}(S, V) ;$ |

Fig. 9 Game AG1
obtained by all the previous queries to $\mathscr{E}$ induced by queries to AEenc or AEdec made by A. For a query to $\mathscr{E}$ or $\mathscr{D}$, let $\mathcal{W}_{\mathrm{BC}}$ be the set of ( $S, U, V$ )'s obtained by all the previous queries to BCenc or BCdec made by A. A query $(S, U)$ to $\mathscr{E}$ is called fresh if $\mathrm{E}[S, U]=\perp$.

Let $\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}$ be the event that, for a fresh query to $\mathscr{E}$ induced by a query to AEenc or AEdec, $\mathscr{E}$ replies $V$ such that $V_{0}=K$ or, for some $\left(S^{\prime}, U^{\prime}, V^{\prime}\right) \in \mathcal{W}_{\mathrm{AE}}, V_{0}=V_{0}^{\prime}$ and $\left\{V_{1}, \pi_{0}\left(V_{1}\right), \pi_{1}\left(V_{1}\right)\right\} \cap\left\{V_{1}^{\prime}, \pi_{0}\left(V_{1}^{\prime}\right), \pi_{1}\left(V_{1}^{\prime}\right)\right\} \neq \emptyset$, that is,

$$
\begin{gathered}
V_{1} \in\left\{V_{1}^{\prime}, \pi_{0}\left(V_{1}^{\prime}\right), \pi_{1}\left(V_{1}^{\prime}\right), \pi_{0}^{-1}\left(V_{1}^{\prime}\right), \pi_{0}^{-1}\left(\pi_{1}\left(V_{1}^{\prime}\right)\right),\right. \\
\left.\pi_{1}^{-1}\left(V_{1}^{\prime}\right), \pi_{1}^{-1}\left(\pi_{0}\left(V_{1}^{\prime}\right)\right)\right\},
\end{gathered}
$$

where $V=V_{0} \| V_{1}, V^{\prime}=V_{0}^{\prime}| | V_{1}^{\prime}$ and $\left|V_{0}\right|=\left|V_{1}\right|=\left|V_{0}^{\prime}\right|=$ $\left|V_{1}^{\prime}\right|=w$. Then,

$$
\operatorname{Pr}\left[\operatorname{Bad}_{\mathrm{AE}}^{\mathrm{AE}}\right] \leq \frac{\sigma}{2^{w}}+\frac{7 \sigma^{2}}{2^{n}-q-\sigma}
$$

Let $\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{BC}}$ be the event that, for a fresh query to $\mathscr{E}$ induced by a query to AEenc or AEdec, $\mathscr{E}$ replies $V$ such that, for some $\left(S^{\prime}, U^{\prime}, V^{\prime}\right) \in \mathcal{W}_{\mathrm{BC}}, S^{\prime}=V_{0}$ and $\operatorname{lsb}_{w}\left(U^{\prime}\right) \in$ $\left\{V_{1}, \pi_{0}\left(V_{1}\right), \pi_{1}\left(V_{1}\right)\right\}$. Then,

$$
\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{BC}}\right] \leq 3 q \sigma /\left(2^{n}-q-\sigma\right)
$$

Let $\mathrm{Bad}_{\mathrm{BCe}}$ be the event that $\mathbf{A}$ makes at least one query $(S, U)$ to BCenc such that $S=K$ or, for some $\left(S^{\prime}, U^{\prime}, V^{\prime}\right) \in$ $\mathcal{W}_{\mathrm{AE}}, S=V_{0}^{\prime}$ and $U_{1} \in\left\{V_{1}^{\prime}, \pi_{0}\left(V_{1}^{\prime}\right), \pi_{1}\left(V_{1}^{\prime}\right)\right\}$. Then, since $q+\sigma \leq 2^{w}$,

$$
\begin{aligned}
\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCe}} \mid \overline{\mathrm{MCol}}\right] & \leq \frac{q_{\mathrm{e}}}{2^{w}}+\frac{2^{w} \cdot 3 \gamma(w) q_{\mathrm{e}}}{2^{n}-(q+\sigma)} \\
& \leq \frac{q_{\mathrm{e}}}{2^{w}}+\frac{3 \gamma(w) q_{\mathrm{e}}}{2^{w}-1}
\end{aligned}
$$

Let $\mathrm{Bad}_{\mathrm{BCd}}$ be the event that $\mathbf{A}$ makes at least one query $(S, V)$ to BCdec such that $S=K$ or, for some $\left(S^{\prime}, U^{\prime}, V^{\prime}\right) \in \mathcal{W}_{\mathrm{AE}}, S=S^{\prime}$ and $V=V^{\prime}$, or $S=V_{0}^{\prime}$ and $\operatorname{lsb}_{w}(U) \in\left\{V_{1}^{\prime}, \pi_{0}\left(V_{1}^{\prime}\right), \pi_{1}\left(V_{1}^{\prime}\right)\right\}$, where $U$ is the reply to the query $(S, V)$. Then,

$$
\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCd}} \mid \overline{\mathrm{MCol}} \cap \overline{\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}}\right] \leq \frac{q_{\mathrm{d}}}{2^{w}}+\frac{q_{\mathrm{d}}}{2^{w}-1}+\frac{3 \gamma(w) q_{\mathrm{d}}}{2^{w}-1}
$$

Let Forge be the event that $\mathbf{A}$ succeeds in forgery. Let Bad $=\operatorname{Bad}_{A E}^{A E} \cup \operatorname{Bad}_{A E}^{B C} \cup \operatorname{Bad}_{B C e} \cup \operatorname{Bad}_{B C d}$. If Bad does not occur, then, since $\left\{\pi_{0}, \pi_{1}, \iota\right\}$ is pairwise everywhere distinct, for each query to AEdec made by $\mathbf{A}$, the final query to $\mathscr{E}$ induced by the query is fresh. Thus,

$$
\operatorname{Adv}_{\text {Lae } 0}^{\text {auth }}(\mathbf{A})=\operatorname{Pr}[\text { Forge }] \leq \operatorname{Pr}[\text { Bad }]+\operatorname{Pr}[\text { Forge } \mid \overline{\mathrm{Bad}}],
$$

where

$$
\operatorname{Pr}[\text { Forge } \mid \overline{\mathrm{Bad}}] \leq q_{\mathrm{D}} /\left(2^{n}-q-\sigma\right)
$$

In addition,

$$
\operatorname{Pr}[\mathrm{Bad}]=\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}} \cup \operatorname{Bad}_{\mathrm{AE}}^{\mathrm{BC}} \cup \operatorname{Bad}_{\mathrm{BCe}} \cup \mathrm{Bad}_{\mathrm{BCd}}\right]
$$

$$
\begin{aligned}
& \leq \operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}\right]+\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{BC}}\right] \\
& \\
& \quad+\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCe}} \cup\left(\mathrm{Bad}_{\mathrm{BCd}} \cap \overline{\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}}\right)\right],
\end{aligned}
$$

and

$$
\begin{aligned}
& \operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCe}} \cup\left(\mathrm{Bad}_{\mathrm{BCd}} \cap \overline{\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}}\right)\right] \\
& \quad \leq \operatorname{Pr}[\mathrm{MCol}]+\operatorname{Pr}\left[\mathrm{Bad}_{\mathrm{BCe}} \cup\left(\mathrm{Bad}_{\mathrm{BCd}} \cap \overline{\mathrm{Bad}_{\mathrm{AE}}^{\mathrm{AE}}}\right) \mid \overline{\mathrm{MCol}}\right] .
\end{aligned}
$$

Thus,

$$
\operatorname{Pr}[\mathrm{Bad}] \leq \frac{3 \gamma(w) q+q_{\mathrm{d}}}{2^{w}-1}+\frac{q+\sigma+1}{2^{w}}+\frac{7 \sigma^{2}+3 q \sigma}{2^{n}-q-\sigma}
$$

This completes the proof.

## 6. Conclusion

The privacy and authenticity of Lae0 have been analyzed in the ideal cipher model. The analysis implies that, for both privacy and authenticity, the instantiation of Lae0 with the Lesamnta-LW block cipher has about 120-bit security against generic attacks regarding the block cipher as a black box.

## Acknowledgments

The first author was supported in part by JSPS KAKENHI Grant Number JP18H05289.

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Shoichi Hirose received the B.E., M.E. and D.E. degrees in information science from Kyoto University, Kyoto, Japan, in 1988, 1990 and 1995, respectively. From 1990 to 1998, he was a research associate at Faculty of Engineering, Kyoto University. From 1998 to 2005, he was a lecturer at Graduate School of Informatics, Kyoto University. From 2005 to 2009, he was an associate professor at Faculty of Engineering, University of Fukui. From 2009, he is a professor at Graduate School of Engineering, University of Fukui. His current interests include cryptography and information security. He received Young Engineer Award from IEICE in 1997, and KDDI Foundation Research Award in 2008.


Hidenori Kuwakado received the B.E., M.E. and D.E. degrees from Kobe University in 1990, 1992, and 1999 respectively. He worked for Nippon Telegraph and Telephone Corporation from 1992 to 1996. From 1996 to 2002, he was a research associate in the Faculty of Engineering, Kobe University. From 2002 to 2007, he was an associate professor in the Faculty of Engineering, Kobe University. From 2007 to 2013, he was an associate professor in Graduate School of Engineering, Kobe University. Since 2013, he has been a professor in Faculty of Informatics, Kansai University. His research interests are in cryptography and information security.


Hirotaka Yoshida received the B.S. degree from Meiji University, Japan, in 1999, the M.S. degree from Tokyo Institute of Technology, Japan, in 2001, and the Ph.D. degree in electrical engineering from KU Leuven, Belgium, in 2013. From 2001 to 2016, he was with the Research \& Development Group, Hitachi, Ltd. He is currently a team leader at the National Institute of Advanced Industrial Science and Technology (AIST). He is a member of IACR, IPSJ, and JSAE. In 2013, he won the award of industrial standardization that has been granted by the Japanese Ministry of Economy, Trade and Industry (METI).


[^0]:    Manuscript received February 9, 2020.
    Manuscript revised June 4, 2021.
    Manuscript publicized July 8, 2021.
    ${ }^{\dagger}$ The author is with Faculty of Engineering, University of Fukui, Fukui-shi, 910-8507 Japan.
    ${ }^{\dagger}$ The author is with Faculty of Informatics, Kansai University, Takatsuki-shi, 569-1095 Japan.
    ${ }^{\dagger \dagger \dagger}$ The author is with National Institute of Advanced Industrial Science and Technology, Tokyo, 135-0064 Japan.
    a) E-mail: hrs_shch@u-fukui.ac.jp
    b) E-mail: kuwakado@kansai-u.ac.jp
    c) E-mail: hirotaka.yoshida@aist.go.jp

    DOI: 10.1587/transinf.2021NGP0008

